

Planckian corrections to the Friedmann flat equations from thermodynamics at the apparent horizon

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Abstract

In this paper we use our recently generalized black hole entropy formula to propose a quantum version of the Friedmann equations. In particular, starting from the differential version of the first law of thermodynamics, we are able to find planckian (non commutative) corrections to the Friedmann flat equations. The so modified equations are formally similar to the ones present in Gauss-Bonnet gravity, but in the ordinary 3+1 dimensions. As a consequence of these corrections, by considering negative fluctuations in the internal energy that are allowed by quantum field theory, our equations imply a maximum value both for the energy density ρ and for the Hubble flow H , i.e. the big bang is ruled out. Conversely, by considering positive quantum fluctuations, we found no maximum for ρ and H . Nevertheless, by starting with an early time energy density $\rho \sim 1/t^2$, we obtain a value for the scale factor $a(t) \sim e^{\sqrt{t}}$, implying a finite planckian universe at $t = 0$, i.e. the point-like big bang singularity is substituted by a universe of planckian size at $t = 0$. Finally, we found possible higher order planckian terms to our equations together with the related corrections of our generalized Bekenstein-Hawking entropy.

Keywords: Friedmann equations; non commutative spacetime; Bekenstein-Hawking entropy; big bang singularity.

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1 Introduction

The Hawking's discovery [1] concerning the radiation from a black hole represents an essential tool to study the relation from thermodynamics, black holes and quantum effects in curved spacetimes. In particular, to a black hole in an asymptotically flat spacetime can be associated an entropy [2] S_{BH} given by $S_{BH} = \frac{k_B A}{4L_P^2}$, where k_B is the Boltzmann constant and L_P the Planck length. This fundamental discovery opened the door to interesting investigations concerning the thermodynamics of the universe as a whole both in general relativity and modified gravity (see for example [3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13]) at the apparent horizon of the universe [14, 15]. Recently (see for example [5, 6, 7, 8, 3, 10, 11, 12, 13]), the interest was devoted to the derivation of Friedmann equations in general relativity and modified gravity from the first law of thermodynamics at the apparent horizon. In all these derivations, thanks to the holographic principle [5, 6], it is assumed for the entropy S_h of the apparent horizon the usual one of a static black hole $S_h = \frac{k_B A_h}{4L_P^2}$, where A_h is the proper area of the apparent horizon. In a cosmological context apparent horizons are dynamical and non static. Moreover, in ordinary cosmology the situation is more involved since we have not at our disposal a way to calculate the dynamical degree of freedom of the gravitational field.

Recently, [16, 17] we proposed a generalized expression for the entropy of black holes embedded in Friedmann universes. This generalized formula can be applied, thanks to the holographic principle, to the apparent horizon and can be interpreted as the total entropy of our universe. In this formula a further term does appear proportional to the Hubble flow H and describing the dynamical degrees of freedom of the gravitational field. As a consequence of this new proposal and according to an old conjecture [18], the classical Friedmann flat universe has a time independent vanishing internal energy. When quantum effects are taken into account by a non vanishing Planck constant \hbar , a series expansion for the internal energy of a Friedmann flat spacetime can be performed.

In this paper, instead of write down the Friedmann equations as a first law at the apparent horizon, starting from these new results and from our proposed first law, we propose a quantum (non commutative) ultraviolet modified version of the Friedmann equations.

In section 1 we review our generalized Bekenstein-Hawking entropy formula for the entropy of the universe. In section 2 we write down our first law for a Friedmann flat spacetime suitable to generalize the Friedmann equa-

tions and in section 3 we present our method applied to classical Friedmann spacetimes. In section 4, we obtain an ultraviolet modification of the de Sitter spacetime and in section 5 we generalize and motivate our method to any Friedmann flat spacetime together with a proposal for a non spatially flat spacetime. Section 6 is devoted to a study of possible higher order planckian corrections. Finally, section 7 is devoted to the conclusions and final considerations.

2 Generalized Bekenstein-Hawking entropy at the apparent horizon

The line element for a Friedmann universe is in comoving coordinates

$$ds^2 = -c^2 dt^2 + a^2(t) \left[\frac{dr^2}{1 - kr^2} + r^2 d\Omega^2 \right], \quad (1)$$

where $k = -1, 0, +1$. A Friedmann universe filled with matter-energy satisfying the weak energy condition is equipped with an apparent horizon at the proper areal radius L_h given by

$$L_h = \frac{c}{\sqrt{H^2 + \frac{kc^2}{a(t)^2}}}. \quad (2)$$

It is customary to associate to the apparent horizon [7] the Hawking temperature $T_h = c\hbar/(2\pi k_B L_h)$. In particular, in the paper [7] we have for the first time a derivation of the Friedmann equation from the first law at the apparent horizon:

$$T_h dS = dU_h + W_h dV_h, \quad W_h = \frac{p - \rho c^2}{2}, \quad (3)$$

where W_h is the work term and $U_h = c^4 L_h/(2G)$ is the Misner-Sharp energy term (see also [17]), p the pressure and ρ the energy density. The Friedmann equations are consistent with (3) provided that it is assumed for the apparent horizon the entropy $S_h = \frac{k_B A_h}{4L_P^2}$. Generally, the entropy of the whole universe at the apparent horizon is assumed to be the sum of the entropy of the holographic screen (S_h) with the usual one of the matter inside L_h . This approach is not absent from criticisms. For example, in [19] it has been shown that there exist different heat sign conventions between the Clausius equation in [7] (positive heat out sign convention) and the usual Gibbs equation (positive heat in sign convention) for the matter inside L_h . This fact

takes necessary a modified version of the first law for the matter content inside L_h . Moreover, as pointed also in [20] at the introduction, we have not at our disposal a way to calculate the dynamical degrees of freedom for a non-static gravitational field. In particular, the assumption that S_h is the entropy suitable also for non static black holes seems trather artificial since we expect a modification caused by the expansion of the universe, i.e. by the Hubble flow H . The same criticism does apply to the usual setup for the entropy inside L_h .

In [16, 17], by using suitable theorems for the formation of trapped surfaces for mass-energy concentration embedded in Friedmann universes, we proposed a generalization for the entropy of a black hole of proper are A_h and volume V_h embedded in a Friedmann universe given by

$$S_h = \frac{k_B A_h}{4L_P^2} + \frac{3k_B}{2cL_P^2} V_h H - \frac{3kk_B}{4L_P^2} \frac{L_h V_h}{a(t)^2}. \quad (4)$$

In the static case, expression (4) reduces to the usual one. To calculate the expression for the entropy of the whole universe at L_h we use the holographic principle [5, 6]. In fact, in the static case, the total entropy for a black hole is given by $S_{ev} = \frac{k_B A_{ev}}{4L_P^2}$, where A_{ev} denotes the proper area of the event horizon, i.e. the entropy of the holographic screen and there is not a contribution from the interior of the black hole. In a cosmological spacetime the event horizon becomes a teleological object that is not attainable for human observers. In this context the useful tool to identify black holes is provided by apparent horizons (see [14, 15] and the discussion in [16, 17]). As a consequence, we can assume that the total entropy (comprising the dynamical degrees of freedom) of the universe is given by the (4) calculated at L_h given by (2). In practice, we suppose that the entropy bound is exactly saturated at the apparent horizon [16, 17]. In the following we mainly consider the flat case with $k = 0$. Concerning the temperature, note that after writing the first law as

$$T_h dS_h = dU_h + W_h dV_h, \quad (5)$$

the equation (5) is left unchanged under a constant β scaling of T_h (i.e. $T_h \rightarrow \beta T_h, U_h \rightarrow \beta U_h, W_h \rightarrow \beta W_h$). We advantage of this to choose $\beta = 1/2$. As a consequence of these setups we have, thanks to the (4) with $k = 0$:

$$dU_h = \frac{c^4}{2G} dL_h + \frac{c^3}{2G} L_h^2 dH, \quad (6)$$

$$W_h = \frac{3c^3 H}{8\pi G L_h}, \quad T_h = \frac{\hbar c}{4\pi k_B L_h}. \quad (7)$$

In this way, with $\beta = 1/2$ the first term in (6) is nothing else but the Misner-Sharp mass and $W_h = \rho$ after using the Friedmann equation $H^2 = (8\pi G\rho)/3$. Also note that after taking $L_h = c/H$ we have $U_h = \text{const}$ ¹. As discussed in [17], this implies $U_h = 0$ in the flat case, i.e. the Friedmann flat universes, according to [18], all have zero internal energy. The fact that only in the spatially flat case we obtain $U_h = 0$ strongly motivates our proposal (4) as the entropy of the universe at its 'thermodynamic' radius L_h . Equations (5)-(7) are the starting point for the investigations of this paper.

3 Friedmann flat equations from the first law of thermodynamics

For a Friedmann flat spacetime the differential first law can be written as

$$T_h dS_h = dU_h + c^2 \rho dV_h, \quad (8)$$

in such a way that only thermodynamic quantities appear in the first law. We assume that the differential form (8) of the first law remains left unchanged also when quantum corrections are taken into account, that is in line with the standard assumptions present in the literature (see for example [7, 8, 11, 12, 13]). We advantage of the presence of ρ in (8) to obtain the field equations by the knowledge of S_h and U_h . To start with, we introduce the free Helmholtz energy $F_h = U_h - T_h S_h$. At the apparent horizon, the proper volume V_h and the temperature T_h are not independent quantities: they satisfy the equality

$$T_h^3 V_h = \frac{1}{48\pi^2} \left(\frac{c\hbar}{k_B} \right)^3. \quad (9)$$

Note that, as pointed in [19], T_h is the temperature of the holographic screen that is generally different from the one T_u of the universe inside L_h . In practice $T_h^3 V_h = \text{const}$, as happens for an isentropic radiation field. As a consequence, we obtain

$$dF_h = -\rho V_{h,T}(T) dT - S_h dT, \quad V_{h,T}(T) = - \left(\frac{c\hbar}{k_B} \right)^3 \frac{1}{16\pi^2 T_h^4}, \quad (10)$$

¹The Misner-Sharp mass in (6) is exactly balanced by the added term describing the dynamic degrees of freedom of the gravitational field.

where ”,” indicates partial derivative. From (10) we can obtain ρ in terms of the total derivative of F_h with respect to T_h , i.e. $\frac{dF_h}{dT_h}$ and S_h . We have

$$c^2 \rho = - \left(\frac{dF_h}{dT_h} + S_h \right) \frac{1}{V_{h,T}}. \quad (11)$$

We can check that, by taking in (11) the expression for S_h dictated by a Friedmann flat universe (equation (4) with $k = 0$)

$$S_h = \frac{3c^5 \hbar}{16\pi G k_B} \frac{1}{T_h^2}, \quad T_h = \frac{\hbar H}{4\pi k_B}, \quad F_h = -T_h S_h, \quad (12)$$

we obtain the classical Friedmann equation

$$\rho = \frac{6\pi k_B^2 T_h^2}{G \hbar^2}. \quad (13)$$

In the next three sections we use this simple technique to generate planckian corrections to the Friedmann equations.

4 Planckian corrections for a cosmological de Sitter universe

With only the constants c, G, Λ (where Λ is the cosmological constant) in the Einstein equations we get [17] $U_h = \text{const} = 0$ for Friedmann flat spacetimes. Conversely, with the introduction of \hbar we can have

$$U_h \sim \frac{G^{n-1} \hbar^n}{c^{3n-4}} \Lambda^{(n-\frac{1}{2})}, \quad n \in N. \quad (14)$$

We can thus build the following series expansion for U_h :

$$\begin{aligned} U_h &= c\hbar\sqrt{\Lambda} [\overline{c}_0 + \overline{c}_2 L_P^2 \Lambda + \overline{c}_4 L_P^4 \Lambda^2 + \cdots + \overline{c}_{2n} L_P^{2n} \Lambda^n + \cdots] \\ &= c\hbar\sqrt{\Lambda} \sum_{n=0}^{\infty} \overline{c}_{2n} L_P^{2n} \Lambda^n, \quad \overline{c}_{2n} \in R. \end{aligned} \quad (15)$$

Note that, apart from the unknown adimensional constants \overline{c}_{2n} , the higher order planckian corrections depend on the product $L_P^{2n} \Lambda^n$ and thus are relevant for $\Lambda \sim 1/L_P^2$. In the following we consider only the first order correction where $U_h = \overline{c}_0 c \hbar \sqrt{\Lambda}$. Section 6 is devoted to the study of higher order planckian corrections. Moreover, to the non vanishing variation $U_h \sim \sqrt{\Lambda}$

can be associated, thanks to (9), a constant entropy (radiation like field) given by

$$S_0 = w_0 \frac{16\sigma}{3c} V_h T_h^3, \quad \sigma = \frac{\pi^2 k_B^4}{60 \hbar^3 c^2}, \quad (16)$$

where $w_0 = 1$ for pure radiation. We have:

$$F_h = 4\pi\sqrt{3}\overline{c_0}k_B T_h - S_0 T_h - \frac{3c^5 \hbar}{16\pi G k_B T_h}, \quad T_h = \frac{c\hbar}{4\pi k_B} \sqrt{\frac{\Lambda}{3}}. \quad (17)$$

From (11) we obtain

$$c^2 \rho_\Lambda = \frac{(4\pi)^3 \overline{c_0} k_B^4 \sqrt{3}}{c^3 \hbar^3} T_h^4 + \frac{3c^2 (4\pi)^2 k_B^2}{8\pi G \hbar^2} T_h^2. \quad (18)$$

Note that, thanks to the modified expression for F_h , a further term proportional to T_h^4 (radiation-like term) does appear in (18). A naive interpretation of (18) suggests a Friedmann equation $H^2 = 8\pi G/3\rho_\Lambda$ with $\rho_\Lambda = \Lambda_{eff} c^2/8\pi G$ and $\Lambda_{eff} = \Lambda + 2\overline{c_0}\sqrt{3}/9\Lambda^2 L_P^2$. Also in modified gravity we expect that (see [8, 11, 12]) $T_h \sim H$, i.e. $T_h \sim \sqrt{\Lambda}$. But with the equation $\rho_\Lambda = \Lambda_{eff} c^2/8\pi G$ we have obviously $T_h \sim \sqrt{\Lambda_{eff}}$, with $\Lambda_{eff} \neq \Lambda$, a contradiction. With this interpretation, backreaction is not taken into account. A self consistent way to read equation (18) is to see this as a modified Friedmann equation. We start with a 'bare' cosmological constant Λ . Thanks to quantum effects depicted by the first term in (15) we have a modified observed cosmological constant $\Lambda_{obs} = \overline{\Lambda}$ with $T_h = \hbar \overline{H}/(4\pi k_B)$:

$$\frac{8\pi G \rho_{\overline{\Lambda}}}{3} = \overline{H}^2 + \frac{2\overline{c_0} L_P^2}{c^2 \sqrt{3}} \overline{H}^4. \quad (19)$$

In practice, thanks to quantum effects we have $\Lambda \rightarrow \overline{\Lambda}$. As customary in the ordinary renormalization group approach (see [21]), the relation between Λ and $\overline{\Lambda}$ is unobservable, but in the modified Friedmann equation (19) $\rho_{\overline{\Lambda}}$ is the measured energy density. The correction term becomes important for a huge $\sim 1/L_P^2$ value of the observed Hubble flow \overline{H} . Our approach is not equivalent to the introduction of an ad hoc holographic running cosmological constant $\Lambda = \Lambda(H)$ coupled with the Hubble flow H , where the usual Friedmann equation $H^2 = 8\pi G/3\rho$ is assumed together with an unusual coupling between matter and geometry². In the next section we write down the equation (19) to generic Friedmann flat spacetimes.

²Although the equation $H^2 = \rho(H)$ is allowed, it represents a conceptual violation of the general relativity where ρ is assigned and its behaviour determines the geometry, namely H . In this regard, the introduction of a density $\rho(H)$ can be better associated to a modification of the Einstein tensor $G_{\mu\nu}$ rather than the energy momentum tensor $T_{\mu\nu}$.

5 Building planckian corrections to Friedmann flat equations

As stated above, for any Friedmann flat spacetime we have $U_h = 0$. For a de Sitter spacetime, also by introducing \hbar with the series (15), we have $U_h = \text{const} \neq 0$. The situation changes for a generic Friedmann flat spacetime. In fact, we can always suppose, in presence of a non vanishing cosmological constant that $U_h \sim \sqrt{\Lambda}$ but now Λ is not proportional to T_h . Fortunately, quantum mechanics comes in help. A certain energy displacement ΔE is allowed provided that during a time Δt we have $\Delta t \Delta E \geq \hbar/2$. By considering again the first term in the expansion (15) we have:

$$4\pi k_B |c_0| \sqrt{3} \Delta t \Delta T_h \geq \frac{\hbar}{2}, \quad (20)$$

where $|c_0|$ allows possible negative fluctuations of a quantum field (see for example [22]). Thanks to the (12) we have

$$|c_0| \sqrt{3} \Delta t \Delta H \geq \frac{1}{2}. \quad (21)$$

Since we find planckian corrections, it is natural to suppose a quantum spacetime [23, 24, 25, 26] at the Planck length L_P . In this context, we can consider physically motivated spacetime uncertainty relations (STUR)³ between time and spatial coordinates. In the Friedmann flat case [28], the chosen coordinates are the cosmic time t and the spatial coordinates $\eta^i = a(t)x^i$, where x^i are the usual cartesian coordinates. Since we consider the universe at its thermodynamic radius L_h , it is natural to consider only localizing states ω_s that have spherical symmetry, in particular at very early times near the big bang where the universe was effectively planckian. In these states ω_s the uncertainty of the coordinates all have the same magnitude, i.e. $c\Delta_s t \sim \Delta_s \eta^1 = \Delta_s \eta^2 = \Delta_s \eta^3 = \Delta_s \eta$. Under these reasonable assumptions the STUR in [28] imply a lower bound for $\Delta_s t$ and $\Delta_s \eta$, namely $c\Delta_s t \sim \Delta_s \eta \sim L_P$. By applying this lower bound in (21) we obtain an upper bound for ΔH given by

$$\Delta_s H_{max} = \frac{c}{2|c_0|\sqrt{3}L_P}. \quad (22)$$

³See [23] for the STUR in minkowski spacetime in the Newtonian approximation, [27] for the STUR in minkowski spacetime without Newtonian approximation and [28] for the generalization to Friedmann flat spacetimes.

Hence, quantum fluctuations are available thanks to the Heisenberg uncertainty relation between time and energy, and coupled to planckian effects we expect an upper limit for the uncertainty of H (and T_h). In fact, the STUR in [28], in presence of an infrared cutoff⁴ imply an upper bound for the mean value of $\omega_s(H)$ given by $\omega_s(H) = \frac{c}{\sqrt{3}L_P}$, in agreement with the (22) with $|c_0| \sim 1/2$. Also note that by taking as infrared limit for the inequality (21) the apparent horizon of our universe, we obtain a lower limit for the Hubble flow that is, at present day and with $|c_0|$ of the order of unity, of the same order of the one effectively measured. This is a known fact in the holographic dark energy context, but in our context can have a new interpretation.

As a consequence of these reasonings, we can use again the expansion (15) for U_h expressed in terms of T_h , with the first term given by $U_h = 4\pi\sqrt{3}c_0k_B T_h$. By considering an observed energy density (including the cosmological constant) depicted by $\sum_i \bar{\rho}_i = \bar{\rho}$ and using exactly the same technique applied for (19) we obtain

$$\frac{8\pi G}{3}\bar{\rho} = \bar{H}^2 + \frac{2\bar{c}_0 L_P^2}{c^2\sqrt{3}}\bar{H}^4. \quad (23)$$

The term $\sim \bar{H}^4$ in (23) is in line with the ultraviolet modifications of the general relativity present in [29] by using the Arnowitt-Deser-Misner formalism and also in brane world scenario (see for example [30]). However, in our approach the modifications of the Friedmann equations arise from quantum effects driven by the thermodynamics at the apparent horizon, while in [29] are a geometrically based consequence of the modification from the onset of the Hilbert-Einstein action. In our approach naturally arises the Planck length indicating a non commutative nature of the ultraviolet modification of the Friedmann equations. Moreover, with our approach we can obtain a physically sound interpretation of the geometric based approaches, as the one in [29], in terms of a variation of the internal energy U_h and the entropy S_h of the universe thanks to quantum fluctuations.

Note that our modified Friedmann equation has the same form of the one present in Gauss-Bonnet gravity. In the Gauss-Bonnet gravity the constant multiplying the term H^4 is vanishing in ordinary $3+1$ dimensions, while our modification is an ultraviolet correction acting on a four dimensional manifold. This formal analogy implies for the other Friedmann flat equation

⁴This infrared cutoff is the particle horizon L_{par} in [28] and the apparent horizon L_h in this paper with $L_{par} \sim L_h$

the following expression:

$$c^2 \left(1 + \frac{4\overline{c_0} L_P^2}{c^2 \sqrt{3}} H^2 \right) H_{,t} = -4\pi G (c^2 \overline{\rho} + \overline{p}), \quad (24)$$

where \overline{p} is the pressure.

Also in [31] an expression similar to (23) was obtained in an attempt to obtain the modified Friedmann equations in Horava-Lifshitz gravity from the first law of thermodynamics at the apparent horizon. The author of [31] obtains this equation that is different from the one present in Horava-Lifshitz modified gravity, deducing that it is not possible to obtain the correct Friedmann expression in Horava-Lifshitz gravity from the first law. We stress again that our derivation of (23) has been obtained from our generalized Bekenstein-Hawking formula without forcing the first law at L_h to be exactly the Friedmann equations with the rather questionable assumption $S_h \sim A_h/4$.

In a quantum context, equations (23) and (24) can be seen as effective field equations where the quantities $\overline{\rho}, \overline{p}, \overline{H}$ are mean values calculated on some quantum state s , i.e. $\overline{\rho} = \omega_s(\rho) = \omega_s(T_{00}), \overline{H} = \omega_s(H)$.

Obviously, a modification of the Friedmann equations implies a modification of our generalized entropy formula (4) $S_h = \frac{k_B A_h}{4L_P^2} + \frac{3k_B}{2cL_P^2} V_h H$. After a simple algebra it is easy to see that the planckian modified expression \overline{S}_h is

$$\overline{S}_h = \frac{k_B A_h}{4L_P^2} + \frac{3k_B}{2cL_P^2} V_h \overline{H} \left[1 + \frac{w L_P^2}{1080\pi c^2} \overline{H}^2 \right]. \quad (25)$$

In section 6 we briefly discuss the role of the other terms in the series expansion (15).

Following the analogy with the Gauss-Bonnet Friedmann equations, we can propose a modification of the non spatially flat Friedmann equations. This could be easily done performing in (23) and (24) the transformation $\overline{H}^2 \rightarrow \overline{H}^2 + \frac{kc^2}{a(t)^2}$. However, at present we have not further arguments to justify this generalization and we leave to future investigations this issue.

From equation (23) we can solve with respect to \overline{H}^2 to obtain:

$$\overline{H}^2 = \frac{c^2 \sqrt{3}}{4\overline{c_0} L_P^2} \left[-1 + \sqrt{1 + \frac{64\pi \overline{c_0}}{3\sqrt{3}} \frac{\overline{\rho}}{\rho_P}} \right], \quad (26)$$

where ρ_P is the Planck density. Obviously, in the limit $L_P \rightarrow 0$ we recover the classical Friedmann equation.

Note that equation (26) is defined for both $\overline{c_0} > 0$ and $\overline{c_0} < 0$. In the next two subsections we study our equations (23) and (24) in the cases $\overline{c_0} > 0$ and $\overline{c_0} < 0$.

5.1 Positive quantum fluctuations

In this subsection we consider the case with $\overline{c_0} > 0$. First of all, we can obtain the behaviour of (26) for low densities $\overline{\rho} \ll (3\sqrt{3}\rho_P)/(64\pi\overline{c_0})$. We have

$$\overline{H}^2 = \frac{8\pi G}{3}\overline{\rho} \left[1 - \frac{\sqrt{3}\pi 2^7 \overline{c_0}}{9} \frac{\overline{\rho}}{\rho_P} \right]. \quad (27)$$

Equation (27) looks like the well known Friedmann flat equation present in loop quantum bouncing cosmologies (see [33] and reference therein). Despite this nice analogy, the equation (26) does not admit a turning point where $\overline{H} = 0$. Moreover, from (26) it is evident that for $\overline{c_0} > 0$ we have not an upper bound for the density and the Hubble flow. Nevertheless, we expect that some planckian effect can arise near the classical big bang singularity. To this purpose, consider the usual setup near the big bang for $\overline{\rho}$: $\overline{\rho} = 3\alpha/(8\pi G t^2)$ for some positive constant α of the order of unity. For $t \rightarrow 0$, the dominant term of (27) it gives:

$$\overline{H} = \frac{\sqrt{B}}{\sqrt{t}}, \quad B = \frac{2\sqrt{3}c^2}{L_P^2\sqrt{\overline{c_0}}} \sqrt{\frac{\alpha}{8\pi G\sqrt{3}\rho_P}}, \quad a(t) = a_0 e^{2\sqrt{B}t}. \quad (28)$$

Although the energy density follows the classical behaviour for $t \rightarrow 0$, we have that $a(t=0) = a_0 \neq 0$. This means that, thanks to quantum effects at planckian level, the point like universe at $t = 0$ is substituted by a spherical planckian but finite ball. This is in agreement with the fact that the STUR in a Friedmann flat spacetime imply, for a spherical localizing state [28] that $\Delta_s \eta \geq L_P$, i.e. a fuzzy sphere. A modification of the nature of the big bang singularity caused by non commutative effects can also be found in [32].

5.2 Negative quantum corrections: big bang is ruled out

In the case $\overline{c_0} < 0$ we expect a stronger manifestation of quantum planckian effects with respect to the case with $\overline{c_0} > 0$. To start with, note that the existence condition for the root in (27) implies a maximum value for $\overline{\rho}$ given by:

$$\overline{\rho}_{max} = \frac{3\sqrt{3}\rho_P}{64\pi|\overline{c_0}|}. \quad (29)$$

The maximum value can be made the one obtained in loop quantum gravity [33], namely $\overline{\rho}_{max} \simeq 0.41\rho_P$ with $|\overline{c_0}| \simeq 0.06$. Obviously, thanks to (27), for

$\bar{\rho}_{max}$ given by (29) we have an \bar{H}_{max} given by

$$\bar{H}_{max} = \frac{c 3^{\frac{1}{4}}}{2\sqrt{|\bar{c}_0|} L_P}. \quad (30)$$

Hence, as in [29], our modified equation (26) with $\bar{c}_0 < 0$ is free from the initial singularity at $t = 0$ and big bang is ruled out. Moreover, equation (24) implies for \bar{H}_{max} a de Sitter solution with $c^2 \rho = -p$. The upper bound in (30) is in agreement with the one present in [28] with \bar{c}_0 of the order of unity. With $|\bar{c}_0| = \sqrt{3}/4$ we have a universe at $t = 0$ given by a ball of proper radius L_P . This interesting result means that non commutative effects with negative fluctuations generates an effective positive cosmological constant that takes the place of the big bang paradigm. Hence, a Friedmann flat universe can be emerged from a minkowski spacetime by quantum fluctuations. The fact that, thanks to our new proposal [16, 17], both the 'unerturbed' (classical) minkowski and Friedmann flat spacetimes have zero internal energy, takes possible this interesting scenario.

As a final remark of this section, consider a universe filled with dark matter $\bar{\rho}_m$ after recombination. We can approximate the (23) with $\bar{c}_0 < 0$:

$$\bar{H}^2 = \frac{8\pi G}{3} \bar{\rho}_m \left[1 + \frac{\sqrt{3}\pi 2^7 |\bar{c}_0| \bar{\rho}_m}{9 \rho_P} \right]. \quad (31)$$

A similar expression can be found in [34]. After dividing the (31) for \bar{H}^2 and introducing the usual density parameter $\bar{\Omega}_m$ we get

$$1 = \bar{\Omega}_m + \bar{\Omega}_m \frac{\bar{\rho}_m}{\rho_P} |\bar{c}_0| \pi 2^7 \frac{\sqrt{3}}{9}. \quad (32)$$

Formula (32) shows that, also away from the primordial planckian era $t \simeq t_p$ and on cosmological scales (apparent horizon), non commutative effects generate an effective cosmological constant $\tilde{\Lambda}$ given by

$$\tilde{\Lambda} = \frac{8\pi G}{c^2} |\bar{c}_0| \frac{\sqrt{3}}{9} \pi 2^7 \frac{\bar{\rho}_m^2}{\rho_P}. \quad (33)$$

Unfortunately, with $|\bar{c}_0|$ of the order of unity, $\tilde{\Lambda}$ is too small to explain the actual value of the cosmological constant. To this purpose a very huge unnatural value for $|\bar{c}_0|$ is necessary. Nevertheless, it is physically interesting that also at infrared cosmological scales planckian quantum effects can exist. Only near the planckian era the non commutative effects are so strong that a huge cosmological constant comes in action with the born of our dynamical universe.

6 Higher order corrections

In this section we study some consequences of possible higher order corrections in the expansion (15). In terms of the temperature T_h we have

$$U_h = 4\pi k_B \sqrt{3\overline{c_0}} T_h + \frac{\overline{c_2} L_P^2}{c^2 \hbar^2} \left(4\pi k_B \sqrt{3}\right)^3 T_h^3 + \dots \frac{\overline{c_{2n}} L_P^{2n}}{c^{2n} \hbar^{2n}} \left(4\pi k_B \sqrt{3}\right)^{2n+1} T_h^{2n+1}. \quad (34)$$

With the same technique of section 4 we obtain

$$\frac{8\pi G}{3} \overline{\rho} = \overline{H}^2 + \frac{2\overline{c_0}}{c^2 \sqrt{3}} L_P^2 \overline{H}^4 + \frac{c_2 6\sqrt{3}}{c^4} L_P^4 \overline{H}^6 + \text{higher orders}, \quad (35)$$

where c_2 is defined in terms of $\overline{c_2}$ and with the entropy term $S_0 T_h$ in (17) substituted with $S_2 T_h^3$ with $S_2 = w_2 \frac{16\sigma}{3c} V_h T_h^3$ where

$$3\overline{c_2} (4\pi \sqrt{3})^3 - \frac{w_2}{270} = 3c_2 (4\pi \sqrt{3})^3. \quad (36)$$

and so on. We obtain an expansion where only even powers of H appear, formally preserving general covariance (see [21] and references therein).

In the case where $\{\overline{c_0}, c_2\} < 0$, similar conclusions outlined in subsection 5.2 are valid. Consider now the case of a generic correction up to $c_{2n} L_P^{2n+2} \overline{H}^{2n+4}$ with $c_{2n} > 0$. By considering again $\overline{\rho} = 3\alpha/(8\pi G t^2)$ we obtain that the term $\sim \overline{H}^{2n+4}$ will be dominant in the limit $t \rightarrow 0$. From (35) we obtain $\overline{H} \sim \frac{1}{t^{\frac{2}{2n+4}}}$ in this limit.

Under the hypothesis that there exists an $n^* \in N$ such that $\forall n > n^*, c_{2n} > 0$ and performing the limit $n \rightarrow \infty$, we have $H \rightarrow \text{constant}$, i.e. a de Sitter universe and once again big bang is ruled out, but this time we must sum all contributions in the series expansion (35). In this case the conclusions of subsection 5.2 does not apply and a small cosmological constant could arise compatible with its small observed value. This can be matter for further investigations.

7 Conclusions and final remarks

In this paper we presented a simple technique to obtain planckian modifications to the Friedmann equations by means of the first law at the apparent horizon we recently presented [16, 17]. We mainly considered the spatially flat case. In this regard, the presence of ρ in (8) permit us to obtain the energy density as a function of the free Helmholtz energy F_h

and S_h , namely equation (11). In the Friedmann flat case [28], our generalized entropy implies $U_h = 0$. However, when quantum fluctuations are considered, a non trivial expression for U_h can be obtained. This non trivial expression for U_h implies a modifications of the Friedmann equations and in turn a generalization of our entropy formula (4). The expression of our modified Friedmann flat equations is very similar to the one obtained in Gauss-Bonnet or Lovelock gravity, but our ultraviolet corrections act on ordinary 3+1 dimensions. Obviously, the modification we obtained of Friedmann flat equations appeared in different contexts in the literature (see for example [29, 30, 34] and references therein), but in our context naturally does appear the Planck length as a measure of the deviation from the classical Friedmann equations, by suggesting a non commutative nature of the ultraviolet corrections. Moreover, the study of section 6 indicates that the ultimate form of the Friedmann equations could be done in terms of a series expansion of even power of \overline{H} .

It is also interesting that our Friedmann equation (26) allows to consider both positive and negative fluctuations in the internal energy U_h . For positive fluctuations and low densities with respect to the Planck one ρ_P , the equation (26) reduces to a form very similar to the one found in loop quantum gravity [33], but the non approximated expression (26) does not allow a maximum for $\overline{\rho}$ and \overline{H} . However, by considering positive fluctuations with a singular $\overline{\rho}$ given by the usual expression $\overline{\rho} \sim 1/t^2$, we found that $a(t) \sim e^{\sqrt{t}}$ for $t \rightarrow 0$. This fact means that also by starting with a singular energy density and Hubble flow, the universe at $t = 0$ has finite Planckian dimensions, and the point-like universe at $t = 0$ is substituted by a finite planckian ball. By considering negative fluctuations, our effective equations imply a maximum allowed value for both $\overline{\rho}$ and \overline{H} that are in agreement with the ones obtained in [28]. In this case big bang is ruled out by non commutative effects that are expected to be dominant for $t \rightarrow 0$. It is also interesting that our equations (23) and (24) with $\overline{\epsilon_0} < 0$ admits a de Sitter phase with $\overline{H} = H_{max}$ when $\overline{\rho} = \rho_{max}$. This is not in agreement with a bouncing non singular cosmology present in loop quantum gravity [33] where ρ_{max} corresponds to $H = 0$. Conversely, this is in agreement with a universe born from a quantum minkowski spacetime. In fact both the unperturbed minkowski and Friedmann flat spacetimes have zero internal energy U_h . Quantum fluctuations depicted in section 4 can thus generate a dynamical Friedmann flat universe with H_{max} and can explain why we live in a Friedmann flat universe, while the bouncing scenario does not solve this fundamental cosmological issue. This fact is certainly matter for future

investigations. Another line of research is to solve equations (23) and (24) to search some cosmological signature or relic provided by non commutative effects.

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